Lesson 26

Recall that we can find the maximum and minimum value of a function of two variables, $z = f(x, y)$, subject to a constraint $g(x, y) = c$ by using the method of Lagrange multipliers:

1. Solve the system of equations $f_x = \lambda g_x$, $f_y = \lambda g_y$, and $g(x, y) = c$ to find points (x, y).
2. Evaluate $f(x, y)$ at all the points found in part (1) to find the maximum and minimum of $f(x, y)$ subject to the constraint $g(x, y) = c$.

1. A rectangular box with a square base is to be constructed from material that costs \$3 per square meter for the bottom, \$1 per square meter for the top, and \$2 per square meter for the sides. Find the largest volume *in* cm^3 that can be constructed for \$108. *(Answer: 27,000,000* cm^3*)*
2. It's currently midnight, and Gloria has a paper due tomorrow at 8am. She knows that currently she will receive 20 out of a total 100 points on the assignment. She estimates her score will improve by $x(20-x)$ points if she writes for x hours, and $y(16-y)$ points if she proofreads for y hours. She also estimates she will lose $(x+y)^2$ points for every hour she continues to work on the assignment past midnight. What is the maximum score she can obtain? *(Answer: 70 points)*
3. Suppose that when heated in a microwave, a circular plate's temperature x inches right of the center and y inches above the center is given by $f(x, y) = y^2 - x^2 + 25$ degrees Celsius. The plate has a radius of 5 inches. What is the hottest temperature along the edge of the plate? *(Answer: 50* ^oC*)*
4. You buy stock in both Google (GOOG) and Yahoo (YHOO). Let x be the value of your Google stock (in hundreds of dollars) and let y be the value of your Yahoo stock (in hundreds of dollars). Suppose x and y are related by the equation: $(x-5)^2 + 16(y-10)^2 = 68$. What is the maximum value of your stocks? (Notice that x and y are in hundreds!) *(Answer: \$2,350)*